2D Vacuum transitions and their holographic interpretation

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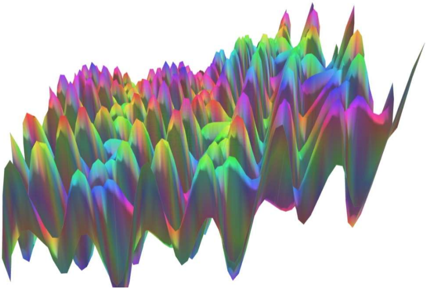
Outline:

1. Motivations
2. methods: Euclidean (CDL, BT), and Hamiltonian (FMP)
3. Holographic interpretation



Motivations

1. populating the String Theory Landscape
2. promising for understanding QG (analogy with BHs)
3. testing the consistency of the 3 formalisms
4. similar unitarity issues as in the information loss paradox
5. analytic calculation in 2D.

1) CDL_2 from JT-gravity using Almheiri-Polchinski

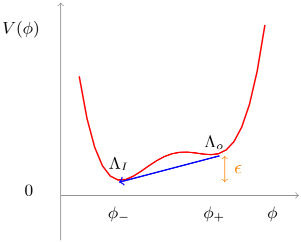
$$S_{2D} = \int d^2x\sqrt{-g}\,\left[\phi^2\mathcal{R} + \lambda(\phi')^2 - U(\phi)\right]$$

$$ds^2 = e^{2\omega}\ \left(dt^2 + dz^2\right) = \rho^2\ \left(dr^2 + r^2 d\theta^2\right)$$

$$S_E = 2\pi\int_0^{\bar{r}} dr\left[\ 2\phi^2\ (r\omega')' + r\ U_o e^{2\omega}\ \right]$$

1. the vacua $(\phi^2)' = 0,\ \partial_{\phi^2} U_o \overset{def}{=} \Lambda = \text{const.}$
2. the wall $(\phi^2)' \neq 0, \Rightarrow B_{wall}$

constraint: $(\phi^2)' = c_1\ r\ e^{2\omega}$



$$\Gamma\ \sim\ \exp\left(iS_o\right) \overset{def.}{=} \exp\left(-B\right)\quad,\quad B \overset{def.}{=} S_E|_{inst} - S_E|_{bckgr}$$

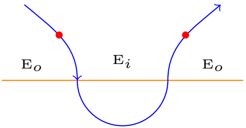
e.g. $$\boxed{B_{tot}^{AdS_2\to AdS_2} = 4\pi\left(\frac{\phi_-^2}{\phi_+^2} + 2\right)\ \left(\phi_-^2 - \phi_+^2\right)}$$

$$B_{tot} \overset{def.}{=} B_{in} + B_{wall}$$

2) BT: brane nucleation (D=d+1), fits well with the Landscape: no V

In 2D : 1. Membrane $\equiv$ particle/anti-particle pair.

2. no magnetic field.



Effectively $$\Lambda_{o,i} \stackrel{def.}{=} \lambda + \frac{1}{2} k \; E_{o,i}^2$$

$$B_{tot}^{type-1}\Big|_{extr} = \frac{4\pi}{k} \ln \left| \frac{r_i}{r_o} \right|$$

$$r_{o,i} \stackrel{def.}{=} \frac{2}{\Lambda_{o,I}\bar{\rho}} \left[1 - \sigma_{o,i} \sqrt{1 - \Lambda_{o,I} \bar{\rho}^2} \right]$$

$$\sigma_{o,i} \stackrel{def.}{=} \mathrm{sgn} \left[\mathcal{E} e E_{on} \mp \frac{1}{4} k \; m^2 \right] \quad , \quad \bar{\rho} \stackrel{def.}{=} \frac{m}{\sqrt{\Lambda_i m^2 + (\mathcal{E} e E_{on} + \frac{1}{4} k \; m^2)^2}}$$

Some preliminary entropic arguments

$$\frac{\Gamma_{t\to f}}{\Gamma_{f\to t}} = \exp\left[-\frac{3}{8}M_P^4\left(\frac{1}{V_t}-\frac{1}{V_f}\right)\right] \qquad \text{[Lee-Weinberg, 1987]}$$

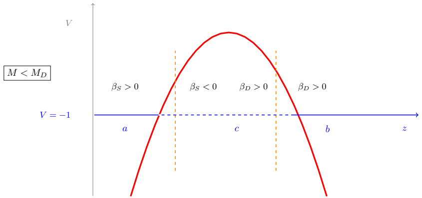
1. Not for $V_t < 0$ due to topology change.
2. $\Rightarrow \quad \frac{\Gamma_{o\to l}}{\Gamma_{l\to o}} = e^{\frac{\eta\pi}{2G}\left(\frac{1}{H_o^2}-\frac{1}{H_l^2}\right)} = e^{\eta(S_o - S_l)} \xrightarrow{\lim_{H_o\to 0}} 0$, since $S_{bckgr} \to \infty$
3. $S_{\mathrm{Sch}\to\mathrm{dS}} = \frac{\eta\pi}{2G}\left[S_{bounce} - S_{bckg}\right] = \frac{\eta\pi}{2G}\left[S_{bounce} - 4G^2M^2\right]$

$\mathrm{Sch}_4 \to \mathrm{dS}_4$ [FGG]

If $M < M_D$

$\lim_{M_S\to 0} \Rightarrow$ brane nucleation

from vacuum fluctuations



1. Avoid the initial singularity problem by quantum tunnelling, but,
2. the instanton is singular. A pseudo-manifold [motivating FMP]

3) The Hamiltonian formulation of VTs [FMP]

1. Lorentzian
2. in 2D using JT.

$$P(A \to B) \overset{def.}{=} \frac{|\Psi(\text{nothing} \to A/B)|^2}{|\Psi(\text{nothing} \to A)|^2}$$

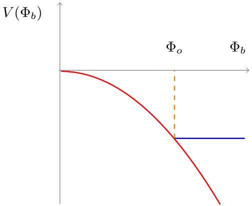
$$g_{\mu\nu} = \begin{pmatrix} -(N^t)^2 + (N^z)^2 & LN^z \\ LN^z & L^2 \end{pmatrix}$$

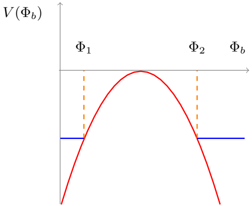
$$\begin{aligned} \mathcal{L}_{2D} &= \sqrt{-g}\,\phi\left[\mathcal{R} - 2\Lambda_+\Theta(z-\hat{z}) - 2\Lambda_-\Theta(\hat{z}-z)\right] + \\ &\quad - \delta(z-\hat{z})\sigma\sqrt{(N^t)^2 + L^2(N^z+\dot{\hat{z}})^2} + \\ &\quad + \sqrt{-g}\left[\phi_o\mathcal{R} - \mathcal{B}_+\Theta(z-\hat{z}) - \mathcal{B}_-\Theta(\hat{z}-z)\right] \\ &= \mathcal{L}_{BTs} + \pi_L\dot{L} + \pi_\phi\dot{\phi} - N^t\mathcal{H}_t - N^z\mathcal{H}_z \end{aligned}$$

$$\mathcal{H}_z = 0\Big|_{z\neq\hat{z}} \Rightarrow \boxed{\pi_L' = \phi'\frac{\pi_\phi}{L}}$$

If $N^t = 1,\ N^z = 0,\ \sigma = \kappa\phi,\ \Phi_b \overset{def.}{=} \frac{\phi_b}{\sqrt{|\mathcal{C}|}}$

$$\begin{cases} -\int_{\hat{z}-\epsilon}^{\hat{z}+\epsilon} dz\ \mathcal{H}_z = 0 \\ -\int_{\hat{z}-\epsilon}^{\hat{z}+\epsilon} dz\ \mathcal{H}_t = 0 \end{cases} \Rightarrow \begin{cases} \left.\frac{\phi'}{L}\right|_{\hat{z}_-}^{\hat{z}_+} = -\frac{\sigma}{2} \\ \left.\pi_L\right|_{\hat{z}_-}^{\hat{z}_+} = 0 \end{cases} \Rightarrow \boxed{\dot{\Phi}_b^2 + V_{eff} = -1}$$





$$ds^2 = \frac{\left(\phi \pm \frac{\mathcal{B}}{2\Lambda_\pm}\right)^2 - 4a^2}{4a^2} \left(-dt^2 + \frac{4a^2}{b^2} \frac{d\phi^2}{\left(\left(\phi \pm \frac{\mathcal{B}}{2\Lambda_\pm}\right)^2 - 4a^2\right)^2} \right)$$

Main results

$$S_{tot}^{\mathrm{AdS}_2\to \mathrm{AdS}_2}\Big|_{\mathcal{B}=0} = \frac{\eta}{G}\left[\mu_- \ln\left|\frac{\mu_- A_1+1}{\mu_- A_1-1}\right| - \mu_+ \ln\left|\frac{\mu_+ A_2+1}{\mu_+ A_2-1}\right|\right] \equiv S_{brane}$$

$$S_{tot}^{\mathrm{dS}_2\to \mathrm{dS}_2}\Big|_{\mathcal{B}=0} = \frac{2\eta}{G}\,\pi\,(\mu_+ - \mu_-) + S_{brane} \xrightarrow{\mu_-\to+\infty} +\infty$$

$$S_{tot}^{\mathrm{Mink}_2\to (\mathrm{A})\mathrm{dS}_2}\Big|_{\mathcal{B}\neq0} = \frac{2\eta}{G}\Big[\phi_f - \phi^-_{h,2} + \rho_2 - \rho_1 +$$

$$+ L_3\left[\tan(h)\left(\frac{\rho_2}{L_3}\right)\ln\left|A_2 \sin(h)\left(\frac{\rho_2}{L_3}\right)\right| - \tan(h)\left(\frac{\rho_1}{L_3}\right)\ln\left|A_2 \sin(h)\left(\frac{\rho_1}{L_3}\right)\right|\right]\Big]$$

Notation: $\mu_\pm \overset{def.}{=} \sqrt{-\frac{\mathcal{C}}{\Lambda_\pm}}$, $A_{1,2} \overset{def.}{=} \frac{\Lambda_+ - \Lambda_- \pm \frac{\kappa^2}{4}}{\kappa}$, $L_3^2 \overset{\text{def.}}{=} \frac{\frac{\mathcal{B}^2}{4\Lambda_-} - \mathcal{C}}{\Lambda_-}$

$$\lim_{\Lambda\to 0} \phi_h\Big|_{\mathcal{B}\neq 0} \overset{def.}{=} \phi_f = \text{const.} \quad , \quad y \overset{def.}{=} \tanh\left(\frac{\rho}{L_3}\right) \overset{\text{def.}}{=} L_3^{-1}\left(\frac{\mathcal{B}}{2\Lambda_-} + \phi\right)$$

BT as $AdS_2/CFT_1 \subset AdS_3/CFT_2$

2 types of Einstein -Maxwell dilaton gravity [Hartman, Strominger, 2008; Alhishahia, , 2008] :

1. type-I $S = \frac{1}{16\pi G}\int dx^2\sqrt{-g}\,\left(e^{\phi}(\mathcal{R}+\frac{2}{\ell^2})+F^2\right)$
2. type-II $S = \frac{1}{16\pi G}\int dx^2\sqrt{-g}\; e^{\phi}\left((\mathcal{R}+\frac{2}{\ell^2})+e^{2\phi}F^2\right)$

Both admit an AdS_2 solution dual to a CFT_1, but only type-II can be obtained by dimensional reduction of AdS_3/CFT_2 where $c \overset{def.}{=} \frac{3R_3}{2G}$.

BT is a type-II theory.

New perspective:

a) looking for a possible entropic understanding of B_{tot} or S_{tot} going beyond detailed balance using holography;

b) possible island interpretation.

Holographic interpretation: examples

$$A) \qquad B_{tot}^{\mathrm{AdS}_2 \to \mathrm{AdS}_2} = 4\pi \left(\frac{\phi_-^2}{\phi_+^2} + 2 \right) \left(\phi_-^2 - \phi_+^2 \right) =$$

$$\stackrel{def.}{=} 4\pi \; S_\pm^2 = 4\pi \; S_+ S_- =$$

$$= 4\pi \; c_{hol}^{CFT_2} \; M_{BH}$$

$$S_\pm \quad \text{generalising} \quad S_{Cardy} = 2\pi \sqrt{\frac{c}{6} \left(\Delta - \frac{c}{24} \right)}$$

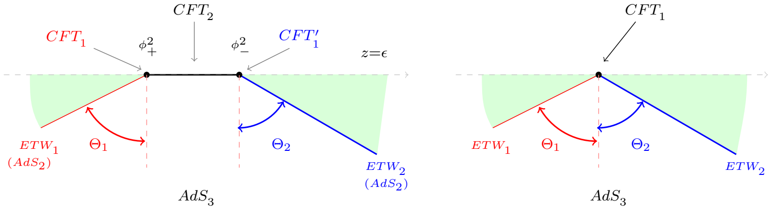
$$c_{hol}^{CFT_2} = \frac{\partial (S_+ S_-)}{\partial \mathcal{Q}} = \left(\frac{\phi_-^2}{\phi_+^2} + 2 \right) \phi_+^2 \quad , \quad M_{BH} = \frac{\phi_-^2}{\phi_+^2} - 1$$

1. the newly-nucleated spacetime is a black hole in the background: relative entropy

2. the spacetimes are extremal and the RG-flow is the CFT_2 with c_{BCFT_2}

3. $\lim_{\Lambda_\pm \to 0} B_{tot} = \infty$ in agreement with the c-theorem.

4. $\phi_-^2 \equiv \phi_+^2 \Rightarrow B_{tot} = 0$ all the information is in the CFT_2 (see next).



after comapctification, it is consistent with wedge holography: $\mathrm{AdS}_2/\mathrm{CFT}_1 \subset \mathrm{AdS}_3/\mathrm{CFT}_2$ just as BT:

$$B) \quad B_{type-1}^{\mathrm{AdS}_2 \to \mathrm{AdS}_2}\Big|_{extr} = 2\pi \left[S_{bdy}^{-} - S_{bdy}^{+} - \ln\left|\frac{\Lambda_+}{\Lambda_-}\right|\right] = 2\pi \; S_{ICFT}$$

with $\quad S_{bdy} \overset{def.}{=} \ln\left|\frac{1+T}{1-T}\right| \quad , \quad T \overset{def.}{=} \sqrt{1-\Lambda_{o,I}\;\bar{\rho}^2}$

Key point: T is uniquely determined by ϵ .

Relation to $T\bar{T}$ $\Rightarrow$ Transitions are local and the CFT$_2$ lives at $z=\epsilon$.
From FMP

$$S_{tot}^{(A)dS\to(A)dS} \;=\; 2\pi\eta\left[\, S^{-}_{T\bar{T}} - S^{+}_{T\bar{T}} \,\right]_{\text{univ.}} \;=\; -2\pi\eta\, F_{\partial} = \; 2\pi\eta\; S_{ICFT}$$

$$-F_{\partial} \overset{\text{def.}}{=} S_{\partial}(R)_{\text{univ.}} \overset{\text{def.}}{=} \lim_{\epsilon\to 0}\left[\,\ln\mathcal{Z}\left[HS^{D}_{R}\right] - \tfrac{1}{2}\ln\mathcal{Z}\left[S^{D}_{R}\right]\,\right]$$

$$S_{T\bar{T}} \overset{def.}{=} \left(1-\frac{r}{2}\frac{\partial}{\partial r}\right)\ln\mathcal{Z}^{(A)dS,\,T\bar{T}}_{S^2} =$$

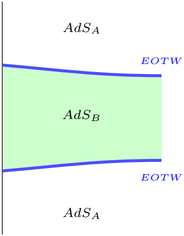
$$= \begin{cases} \pi\frac{c}{3}\sinh^{-1}\left(\sqrt{\frac{12}{c\lambda}}\,r\right), & \text{[H.Verlinde et al.]} \\ \frac{c}{6}+\frac{c}{3}\;\ln\left|\sqrt{\frac{\lambda c}{12}}\frac{1}{r}\right| + \frac{c}{3}\;\tan^{-1}\left(\frac{1}{\sqrt{\frac{\lambda c}{12r^2}-1}}\right) + \text{const.}, & \text{[Silverstein et al.]} \end{cases}$$

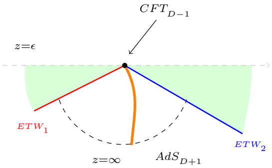
$$\text{with}\quad \mu \overset{def.}{=} \sqrt{\frac{\lambda c}{12}}\;,\quad \phi_o \overset{def.}{=} r\;,\quad \lambda \overset{def.}{=} 2\sqrt{-\frac{\mathcal{C}}{\Lambda}}$$

The emergence of the island [equivalent to Van Raamsdonk et al.]

$\mathrm{AdS_2/CFT_1} \subset \mathrm{AdS_3/CFT_2}$

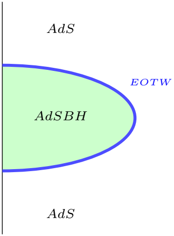
$$S_{tot}^{\mathrm{AdS_2}\to\mathrm{AdS_2}}\Big|_{\mathcal{B}=0} \equiv$$

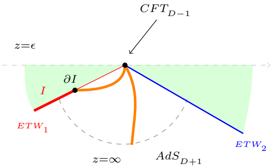




$\mathrm{AdS_2/CFT_1} \not\subset \mathrm{AdS_3/CFT_2}$

$$S_{tot}^{\mathrm{AdS_2}\to\mathrm{AdSBH_2}}\Big|_{\mathcal{B}\neq 0} \equiv$$





$$\boxed{S_{brane} = S^{(2)}_{EE}\Big|_{\mathcal{B}=0} + \delta S_{EE}^{\ s} + \delta S_{EE}^{\ t} + \mathcal{O}(\mathcal{B}^2)} \quad , \quad \phi_2 \overset{def.}{=} -\frac{\mathcal{B}}{|\mathcal{C}|}\phi_o^2 - \phi_1$$

Phase transition beyond a certain value of $\mathcal{B}$.

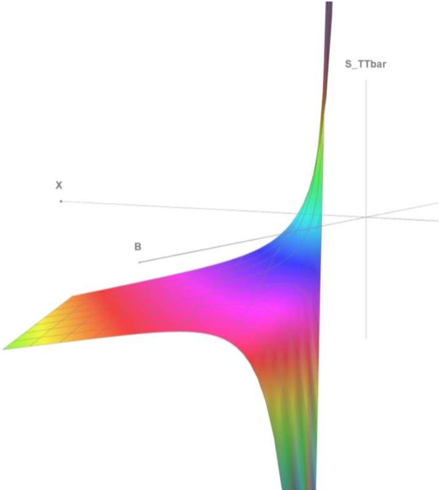
$$S_{brane} = \eta\left(S^{T\bar{T}}_{EE,-} \pm S^{T\bar{T}}_{EE,+}\right)$$

$$S^{1,2}_{EE} = S^{(2)}_{EE}\Big|^{1,2}_{\mathcal{B}=0} + \delta S_{EE}^{\ s}\Big|^{1,2}$$

$$\sim -\frac{8c_-}{3}\ln\left|\frac{1}{\epsilon}\sinh\left(\frac{2\pi r_-}{\beta_-}\right)\right| +$$

$$-\left(1-\frac{\phi_o^2}{\mu_-^2}\right)\frac{\mathcal{B}c_-^2}{9\Lambda_-}\coth^2\left(\frac{2\pi r_-}{\beta_-}\right) +$$

$$+\frac{\frac{c_-}{3}}{\sinh^2\left(\frac{2\pi r_-}{\beta_-}\right)}$$



	AdS_2	dS_2	$Mink_2$
AdS_2	CDL BT FMP	CDL FMP	CDL FMP
dS_2	CDL BT FMP	CDL BT FMP	CDL BT FMP
$Mink_2$	CDL BT FMP	CDL FMP	

Main results:

1. Agreement in between the 3 formalisms in most of the cases, thanks to their holographic interpretation

2. The flat limit of 2D vacuum transitions requires a black hole

3. black holes play a similar role as $T\bar{T}$-deformations in a CFT_2

4. the BH mass is related to ϵ_{UV} of a CFT_2

5. the behaviour of the total action is similar to the difference of 2 generalised entropies [Maldacena et al.]

$$S_{tot}^{\text{(A)dS}\to\text{(A)dS}} = 2\pi\eta \left[\frac{\phi_{h,+}}{G} - \frac{\phi_{h,-}}{G} + S_{EE,-}^{(A)dS,T\bar{T}} - S_{EE,+}^{(A)dS,T\bar{T}}\right]$$

Future directions & Work in progress:

1. extension to SUGRA solutions
2. ...